Impact of Random Fabrication Errors on Fundamental Forward-Wave Small-Signal Gain and Bandwidth in Traveling-Wave Tubes With Finite-Space-Charge Electron Beams

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***Abstract*—The 1-D small-signal theory for the fundamental spatial harmonic mode developed by Pengvanich *et al.* is adapted to include the effect of space charge forces in the electron beam. This model allows us to look at how traveling-wave tube (TWT) performance is affected by random fabrication errors, which are modeled as random perturbations of the phase velocity, interaction impedance, and loss along the TWT's length. In particular, we examine the effect on TWT gain and instantaneous 1-dB bandwidth. Random variation of the phase velocity is found to have the largest effect on both the gain and bandwidth, but the impact is reduced as the amount of space charge in the beam is increased.**

***Index Terms*—Fabrication error, manufacturing tolerance, space charge, traveling-wave tube (TWT).**

I. Introduction

VACUUM ELECTRONIC devices couple energy between electromagnetic waves and an electron beam using electromagnetic waveguides, cavities, and slow-wave structures (SWSs) that have sizes proportional to their operational radio-frequency (RF) wavelength. Consequently, as operating frequencies move toward the millimeter-wave (mm-wave) and terahertz (THz) regimes (wavelengths approximately 1 to 0.1 mm), the size of these SWS features must be reduced accordingly. With nominal feature sizes smaller than a millimeter, the dimensional accuracy and precision required to fabricate these structures become increasingly stringent. A single fabrication error on the order of even a few micrometers can represent a significant deformation to a structure of this size.

Pengvanich *et al.* [1] have recently explored how random fabrication errors (encoded as random perturbations of the Pierce parameters along the length of the device) affect the fundamental forward-wave mode gain and insertion phase of a traveling-wave tube (TWT). They showed that, for errors with magnitudes that are relevant for mm-wave devices, a linear relationship between the standard deviation of the fabrication error and the standard deviation of both the output gain and phase is expected. This trend was observed for independent random errors in circuit phase velocity, impedance, and loss, although phase velocity errors were found to have the largest impact on TWT performance.

More recently, Chernin *et al.* [2] have similarly explored the effect of random fabrication errors on TWT gain and insertion phase but included the effect of multiple internal RF reflections from the arbitrarily positioned discontinuities (i.e., fabrication errors). They observed that both the gain and phase of the TWT were more significantly impacted than what was originally reported in [1]. They also observed that these reflections could cause significant gain ripple versus frequency.

We have built on the theory in [1] by adding finite space charge to the model. This is of general interest since modern TWTs typically utilize electron beams that have nonnegligible space charge [3]. This is particularly true for mm-wave and THz-regime devices [4], [5]. We seek to understand if the relationship between the magnitude of the fabrication errors and the standard deviation of the output gain remains linear for these devices. Additionally, we have extended the analysis by looking at how the instantaneous 1-dB gain bandwidth is affected when random fabrication errors are introduced.

II. Theory

The derivation of the model follows that in [1] very closely but includes the effect of space charge on the electron beam. In doing so, the "electronic equation," which describes the motion of the electrons in the TWT, is given as

$$\left[\left(\frac{\partial}{\partial z}+j\beta_e\right)^2+\beta_q^2\right]s=a \tag{1}$$

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where $\beta_e \equiv \omega/u_0$ is the wavenumber associated with the electron beam disturbance, ω is the operational frequency of the TWT, and u_0 is the dc velocity of the electron beam. The parameter $\beta_q \equiv \omega_q/u_0$ is the reduced space charge wavenumber, where $\omega_q \equiv R_{\rm sc}\omega_p$, $R_{\rm sc}$ is the plasma frequency reduction factor [6]–[8] in which the beam's plasma frequency is given by $\omega_p^2 \equiv e\rho/(m\epsilon_0)$, ρ is the volume charge density of the beam, e and m are the charge and rest mass of an electron, respectively, and ϵ_0 is the permittivity of free space. The variable $s = \tilde{i}_1'$ is a variable substitution for the ac beam current used to make the nomenclature match that used in [1]. The parameter $a \equiv -je\lambda_0\beta_e^2\tilde{E}_z'/(m\omega)$ is the normalized electric field acting on the electron beam element, where λ_0 is the linear charge density of the beam and $\tilde{E}_z'$ represents the complex magnitude of the axially directed electric field of the RF wave propagating along the SWS. MKS units were used for all variables. The only difference between (1) and [1, eq. (1)] is the presence of the β_q^2 space charge term.

$R_{\rm sc}$ is introduced to acknowledge that the space charge force term is not only linearly related to the charge density in the electron bunches of the beam but also limited by the fact that the electron beam is finite in diameter and in close proximity to image charges in nearby conducting surfaces. The value of $R_{\rm sc}$ does not impact the conclusions that can be drawn from the model however. Instead of attempting to use accurate predictions for the value of $R_{\rm sc}$, we opt to vary the value of the normalized Pierce parameter for space charge $4QC$, which, by definition, includes $R_{\rm sc}$. The determination of $R_{\rm sc}$ is beyond the scope of this work.

In addition to the electronic equation, the "circuit equation" is also defined, which describes the RF wave on the SWS induced by the beam's modulated current. For this model, the circuit equation is identical to [1, eq. (2)] and is given as

$$\left(\frac{\partial}{\partial z} + j\beta_p + \beta_p C d\right) a = -j(\beta_e C)^3 s \tag{2}$$

where $\beta_p \equiv \omega/v_p$ is the "cold" circuit wavenumber of the RF field as it propagates on the SWS without the beam present, v_p is the cold circuit phase velocity, and $C^3 \equiv I_0K/(4V_0)$ is the dimensionless Pierce gain parameter, where I_0 is the dc electron beam current, V_0 is the beam voltage corrected for space charge depression, and $K \equiv |E_z|^2/(2\beta_p^2P_z)$ is the Pierce interaction impedance of the circuit, where E_z is the axial component of the RF wave's electric field and P_z is the total RF power flowing along the SWS as given by Poynting's theorem. The parameter d is the unitless circuit loss parameter.

By invoking Ramo's theorem [9], we assume that the modulated electron current in the electron beam induces equivalent currents in the SWS. This allows (1) and (2) to be combined to form a single third-order differential equation. This is simplified by first defining a normalization variable f such that

$$s = e^{-j\beta_e z} f(\beta_e z) = e^{-jx} f(x) \tag{3}$$

where $x \equiv \beta_e z$ is the phase length along the SWS. Combining (1)–(3) gives

$$\frac{d^3f(x)}{dx^3} + jC(b - jd)\frac{d^2f(x)}{dx^2} + C^2(4QC)\frac{df(x)}{dx} + jC^3\left[(4QC)(b - jd) + 1\right]f(x) = 0 \tag{4}$$

where $b \equiv (\beta_p - \beta_e)/(\beta_e C) = (u_0 - v_p)/(v_p C)$ is the Pierce velocity detuning parameter and $4QC \equiv [\beta_q/(\beta_e C)]^2 = [R_{\rm sc}^2(\omega_p^2/\omega^2)/C^2]$ is the Pierce space charge parameter.

Next, the initial conditions for (4) are defined at the input of the device $z = 0$, which corresponds to the phase length $x = 0$. From (3), $f(x)$ is proportional to the ac beam current s which is assumed to be zero at $x = 0$. The parameter $f'(x) = df(x)/dx$, which is proportional to the ac velocity of the electron beam element assuming a time harmonic solution, is also assumed to be zero at $x = 0$. Finally, using (1) and (3), it can be shown that $f''(x) + C^2(4QC)f(x)$ is proportional to the normalized axial RF electric field a, where $f''(x) = d^2f(x)/dx^2$. Since the magnitude of the electric field at the input of the TWT is an arbitrary constant and since $f(0) = 0$, we set $f''(0) = 1$. In summary, at the TWT input $x = 0$, the initial conditions of the differential equation are given as

$$\begin{aligned} f(0) &= 0 \\ f'(0) &= 0 \\ f''(0) &= 1. \end{aligned} \tag{5}$$

Finally, the power gain of the fundamental forward-wave mode is defined as

$$\begin{aligned} \text{Gain}_{\rm FW} &= \left|\frac{f''(x) + C^2(4QC)f(x)}{f''(0) + C^2(4QC)f(0)}\right|^2 \\ &= \left|f''(x) + C^2(4QC)f(x)\right|^2. \end{aligned} \tag{6}$$

For constant and uniform b, d, and $4QC$, (4) can be reduced to the determinantal equation originally given by Pierce [10, p. 113] if a spatial harmonic solution is assumed. That is, if a and s are assumed to vary as $e^{j\beta z}$, (4) can be reduced to

$$\xi^2 + \frac{1}{(\xi - b + jd)} - 4QC = 0 \tag{7}$$

where the RF wavenumber in the presence of the electron beam β is assumed to differ from the electron beam's wavenumber β_e by a small amount ξ (i.e., $\beta = \beta_e + \beta_e C\xi$, where $|C\xi| \ll 1$). Equation (7) is identical to Pierce's determinantal equation, although Pierce used $j\delta$ instead of ξ.

III. RESULTS

Equation (4) was solved subject to the boundary conditions defined in (5) while the Pierce parameters b, C, and d were simultaneously allowed to independently vary randomly along the length x of the simulated TWT. We did not explore the direct effect of independent random variations of the space charge parameter $4QC$ however. This effect was neglected

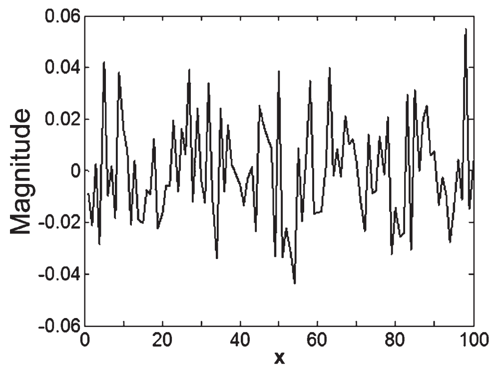

Fig. 1. Example of a piecewise linear function used to simulate the randomly varied Pierce parameters. For this example, the mean was set to zero and the standard deviation was set to 0.02.

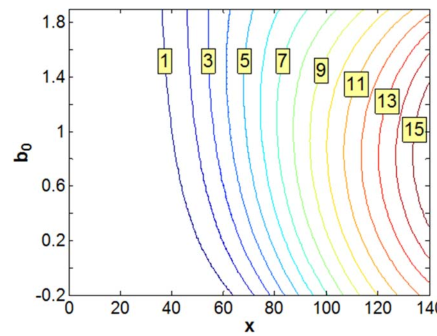

Fig. 2. Calculated fundamental forward-wave mode gain averaged over 1000 trials (each trial having a different random distribution of phase velocity errors) versus axial phase length x and Pierce velocity parameter b_0. In this example, we used $C = 0.028$, $4QC = 0.9$, $d = 0$, and a phase velocity variation of $\sigma_q = 5\%$. The values specified in the boxes on the contour lines represent the fundamental forward-wave mode gain on that line in decibels.

since random fabrication errors are expected to induce only a small negligible variation of the space charge parameter.

Similar to that in [1], the Pierce parameters were assumed to be piecewise linear functions along the TWT's length. For this series of simulations, we assumed a TWT phase length of $x = 100$ and defined the randomly varied Pierce parameters at $x = 1, 2, 3, \ldots, 100$. By (3), this corresponds to fabrication errors with a correlation length of $1/\beta_e$. Assuming that a typical mm-wave TWT operates at a frequency ranging from 30 to 100 GHz and has a beam voltage between 10 and 20 kV, a phase length of $x = 100$ corresponds to a physical length of approximately 0.9–4.3 cm. These frequencies, beam voltages, and physical lengths correspond well with previously published mm-wave TWT designs [11]–[15].

The Pierce parameters at each of these "nodes" were assumed to be independent Gaussian random variables with specified means and standard deviations. An example of the piecewise linear function is shown in Fig. 1. We denote the mean value as μ and the standard deviation as σ. For example, we specify the velocity parameter as $b = b_0 + b_1$, where $b_0 = \mu_b$ is the mean value and b_1 is a Gaussian random number defined by the standard deviation σ_b. Only one Pierce parameter was allowed to vary during a given simulation. It is acknowledged that random fabrication errors would likely affect all the Pierce parameters simultaneously, but by simulating the effects independently, it is easier to identify which error modality has the greatest impact on TWT performance [1].

For each specific standard deviation, the calculation of (4) was repeated 1000 times. The Pierce parameters were independently randomized along the length of the TWT for each trial, giving a different piecewise linear function for each. The results did not significantly differ when 500 or 2000 trials were simulated.

For each trial, the gain was calculated at each phase length x according to (6). Over the ensemble of trials, the mean and standard deviation of the gain at each phase length x were calculated, and the maximum mean gain and the corresponding length were determined and stored. This process was then repeated over a band of b_0 values. Assuming a constant electron beam voltage, varying b_0 equates to varying the phase velocity (and thereby synchronism) of the RF wave. Since b_0 is directly related to the beam/RF synchronism and the RF phase velocity is proportional to ω, we calculated the full-width 1-dB bandwidth of the maximum mean gain with respect to b and used it as an indicator of the effect of errors on instantaneous spectral bandwidth. Additionally, from traditional Pierce theory (i.e., without fabrication errors), increasing $4QC$ results in the maximum gain occurring at a higher b value [3]. Therefore, it was necessary to sweep the calculation versus b_0 in order to not only calculate the bandwidth but also ensure that the maximum gain was determined for each $4QC$ value tested.

In order to maintain consistent results as b_0 was varied, the piecewise linear function representing the Pierce parameters remained constant for all b_0 values in a given trial. In other words, although the Pierce parameters were randomly varied along x for a given trial, they remained constant as b_0 was varied. In doing this, the effects of fabrication errors were assumed to be frequency independent for a given trial. Modeling frequency dependence is left for future study.

A. Random Variation of the RF Phase Velocity

Random fabrication errors in the construction of a TWT can result in random variation of the RF phase velocity along its length. We described these errors using $q(x) \equiv (v_q(x) - v_{q0})/v_{q0}$, where v_{q0} is the unperturbed RF phase velocity (i.e., the phase velocity of the RF wave on the SWS without fabrication errors present), similar to that in [1]. The relationship between σ_q and the standard deviation of the Pierce velocity parameter σ_b is given by $\sigma_b = (\sigma_q/C)(1 + Cb_0)$ [1].

An example of the calculation of the fundamental forward-wave mode gain versus the phase length x and the mean Pierce velocity parameter b_0 is given in Fig. 2. Each (x, b_0) point represents the averaged gain over the 1000 trials simulated. Thus, the figure represents 2 800 000 individual simulations (1000 trials each with 140 x positions and 20 b_0 positions). In this example, $x = 0–140$, $C = 0.028$, $4QC = 0.9$, $d = 0$, and $\sigma_q = 0.05$. Therefore, a lossless TWT was simulated with a finite amount of space charge and a random phase velocity error with a standard deviation of $\pm 5\%$. As shown in Fig. 2, the resultant fundamental forward-wave mode gain has a maximum of approximately 15.8 dB near $b_0 = 0.9$ and $x = 140$.

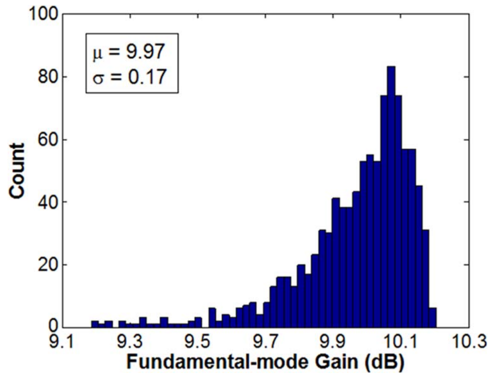

Fig. 3. Distribution of maximum calculated fundamental forward-wave mode gain at $x = 100$ for $C = 0.028$, $4QC = 0.9$, and $d = 0$. A 5% phase velocity error σ_q was simulated. The calculated Gaussian mean μ and standard deviation σ of the distribution are given in the upper left corner.

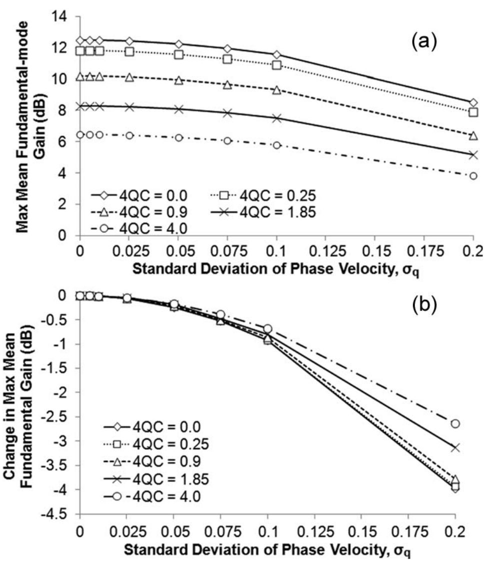

Fig. 4. Variation of maximum mean fundamental forward-wave mode gain at $x = 100$ with $C = 0.028$ and $d = 0$ at different $4QC$ values as a function of the magnitude of the random SWS phase velocity errors. The unnormalized results presented in (a) are normalized to the "error-free" case for each $4QC$ value in (b).

Further examination of the distribution of the calculated gain over the 1000 simulated trials was completed by plotting the distribution of the *maximum* gain (with respect to b_0) at a fixed axial length; in this case, $x = 100$. The results are shown in Fig. 3. Although this distribution is not precisely Gaussian in nature, Gaussian statistics were used to approximately describe the mean and standard deviation of the gain distributions since they are widely understood and allowed for easy comparison with that in [1] which also used Gaussian statistics. A better fit statistical distribution may be more interesting for subsequent studies but is beyond the scope of this analysis.

Fig. 4(a) and (b) shows the absolute and relative variation of the maximum mean fundamental forward-wave mode gain at $x = 100$ as a function of the magnitude of the phase velocity error at different $4QC$ values. The values $C = 0.028$ and $d = 0$ were used. Fig. 5 shows the corresponding standard deviation. Fig. 6 gives the calculated full-width instantaneous 1-dB band-

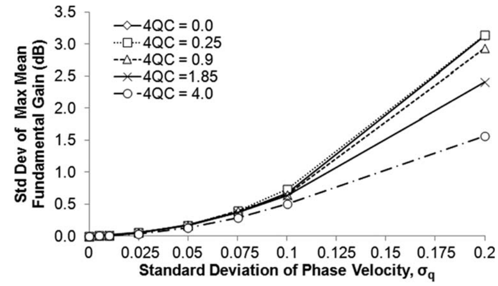

Fig. 5. Standard deviation of maximum mean fundamental forward-wave mode gain at $x = 100$ with $C = 0.028$ and $d = 0$ at different $4QC$ values as a function of the magnitude of the random SWS phase velocity errors.

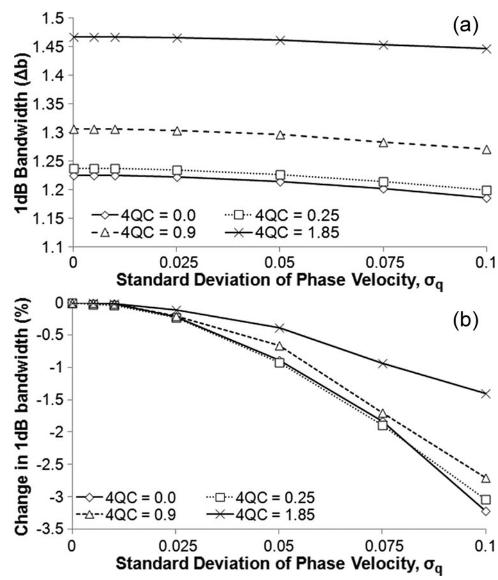

Fig. 6. Variation of full-width 1-dB bandwidth at $x = 100$ with $C = 0.028$ and $d = 0$ at different $4QC$ values as a function of the magnitude of the random SWS phase velocity errors. (a) shows the unnormalized results, while (b) shows the percentage change from the "error-free" case for each $4QC$ value.

width of the maximum mean gain in terms of b for the same series of simulations.

B. Random Variation of the Interaction Impedance

We next considered random variations of the Pierce gain parameter C. Since the cube of the gain parameter is linearly proportional to the interaction impedance of the circuit, this is equivalent to testing the effect of random variations of the interaction impedance. Again, we described the random variation as $C^3(x) = C_0^3[1 + p(x)]$, where C_0 is the unperturbed Pierce gain parameter (i.e., the value of C that corresponds to the SWS without fabrication errors present) and $p(x)$ is a normally-distributed spatially-dependent random number having a standard deviation of σ_p, similar to the analysis completed in [1]. The correlation between σ_p and σ_C is given as $\sigma_C = C_0(\sigma_p/3)$ [1].

During these simulations, the value of the space charge parameter $4QC$ was self-consistently calculated as C was varied since $4QC$ depends intrinsically on C. We assumed that the electron beam parameters (i.e., voltage, current, and charge density) remained constant for each simulation; therefore, the

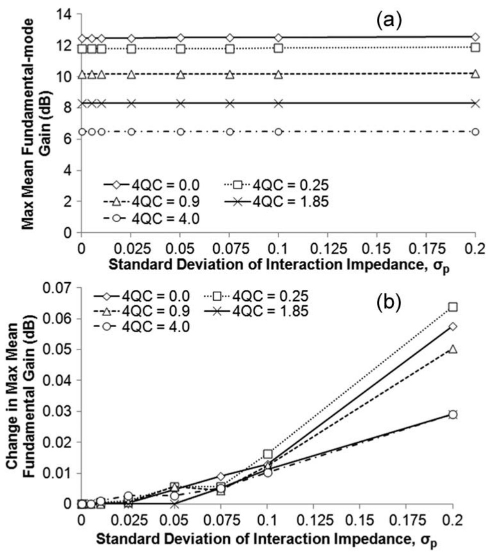

Fig. 7. Variation of maximum mean fundamental forward-wave mode gain at $x = 100$ with $C_0 = 0.028$ and $d = 0$ at different $4QC$ values as a function of the magnitude of the random interaction impedance errors. The unnormalized results presented in (a) are normalized to the "error-free" case for each $4QC$ value in (b).

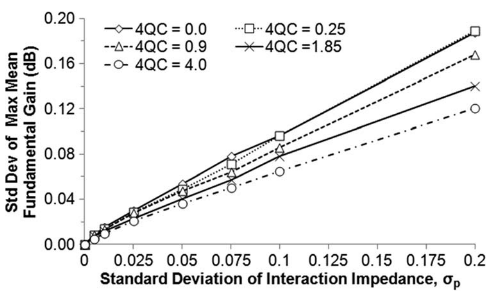

Fig. 8. Standard deviation of maximum mean fundamental forward-wave mode gain at $x = 100$ with $C_0 = 0.028$ and $d = 0$ at different $4QC$ values as a function of the magnitude of the random interaction impedance errors.

change in $4QC$ is given as $4QC(x) = [C_0^2/C^2(x)]4QC_0$, where $4QC_0$ is the unperturbed space charge parameter.

Fig. 7(a) and (b) shows the absolute and relative variation of the maximum mean fundamental forward-wave mode gain at $x = 100$ as a function of the magnitude of the Pierce gain parameter error at different $4QC$ values. The values $C_0 = 0.028$ and $d = 0$ were used. Fig. 8 shows the corresponding standard deviation. Fig. 9 gives the calculated full-width instantaneous 1-dB bandwidth of the maximum mean gain with respect to b for the same series of simulations.

C. Random Variation of the RF Loss

Finally, we considered random variation of the Pierce loss parameter d along the length of a TWT. Again, we described this perturbation as $d(x) = d_0 + d_1$, where d_0 is the unperturbed Pierce loss parameter and d_1 is a Gaussian random number defined by the standard deviation σ_d, similar to the analysis completed in [1]. Since only positive values of d represent RF loss in the simulation, it was necessary to set $d_0 = 0.5$ and limit $3\sigma_d < 0.5$.

Fig. 10(a) and (b) shows the absolute and relative variation of the maximum mean fundamental forward-wave mode gain at $x = 100$ as a function of the magnitude of the Pierce loss parameter error at different $4QC$ values. The values $C = 0.028$ and $d_0 = 0.5$ were used. Fig. 11 shows the corresponding standard deviation. Fig. 12 gives the calculated full-width instantaneous 1-dB bandwidth of the maximum mean gain with respect to b.

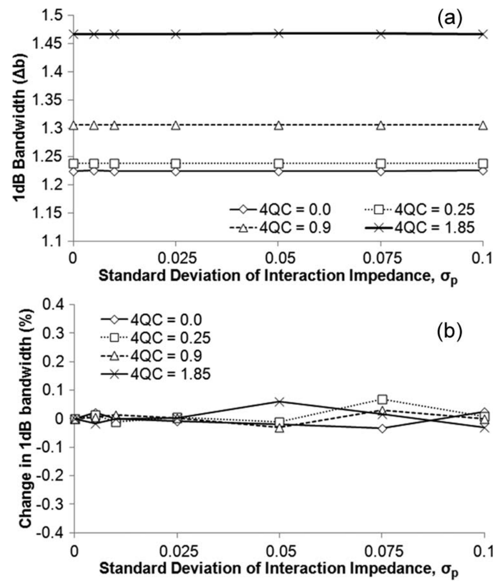

Fig. 9. Variation of full-width 1-dB bandwidth at $x = 100$ with $C_0 = 0.028$ and $d = 0$ at different $4QC$ values as a function of the magnitude of the random interaction impedance errors. (a) shows the unnormalized results, while (b) shows the percentage change from the "error-free" case for each $4QC$ value.

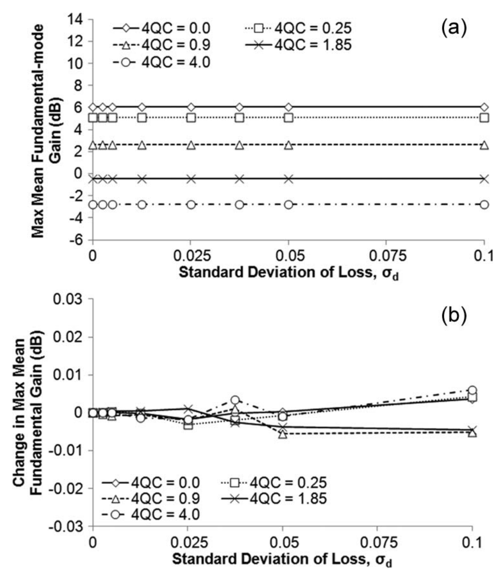

Fig. 10. Variation of maximum mean fundamental forward-wave mode gain at $x = 100$ with $C = 0.028$ and $d_0 = 0.5$ at different $4QC$ values as a function of the magnitude of the random RF loss errors. The unnormalized results presented in (a) are normalized to the "error-free" case for each $4QC$ value in (b).

IV. Discussion

We observe from comparing Figs. 4, 5, 7, 8, 10, and 11 that random variations of the phase velocity have the greatest impact

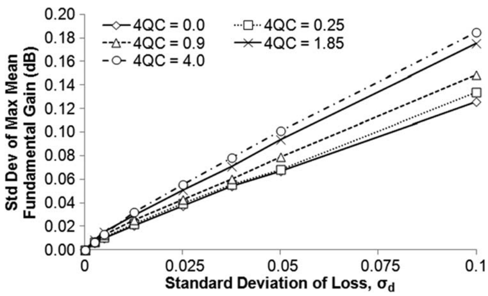

Fig. 11. Standard deviation of maximum mean fundamental forward-wave mode gain at $x = 100$ with $C = 0.028$ and $d_0 = 0.5$ at different $4QC$ values as a function of the magnitude of the random RF loss errors.

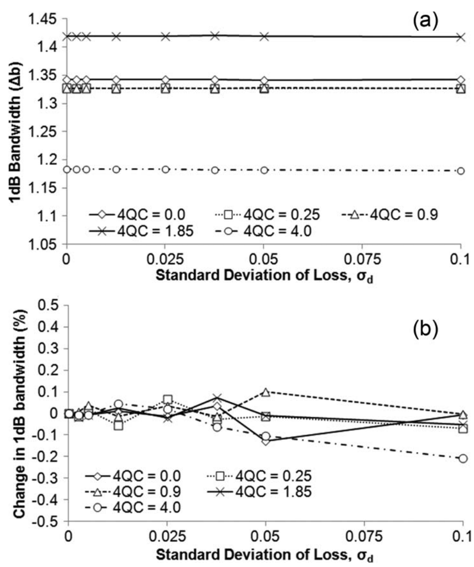

Fig. 12. Variation of full-width 1-dB bandwidth at $x = 100$ with $C = 0.028$ and $d_0 = 0.5$ at different $4QC$ values as a function of the magnitude of the random RF loss errors. (a) shows the unnormalized results, while (b) shows the percentage change from the "error-free" case for each $4QC$ value.

on TWT performance, similar to the conclusion drawn in [1]. This is true regardless of the level of space charge present in the electron beam. From these figures, a 10% random error in the SWS interaction impedance or loss corresponds to an absolute change of the mean fundamental forward-wave mode gain of less than 0.02 dB and a standard deviation of less than 0.2 dB, but a 10% random variation of the phase velocity corresponds to an absolute change of the mean gain of up to 1 dB with a standard deviation of up to ∼0.8 dB. We conclude that fabrication errors that significantly affect the phase velocity of the RF wave on the SWS are the most critical to control, consistent with the observations published in [1], [2], and [16].

By focusing solely on phase velocity errors, we observe that, although increasing the amount of space charge does lower the "error-free" fundamental forward-wave mode gain, it also lessens the effect of errors. For our specific example, increasing $4QC$ from zero to four lowers the "error-free" fundamental forward-wave mode gain from 12.5 to 6.5 dB, but it also decreases the change in the mean fundamental forward-wave gain that occurs when a 20% random phase velocity error is introduced from -4 to -2.6 dB, as shown in Fig. 4. We simultaneously observe a decrease of the standard deviation of the fundamental forward-wave gain at $x = 100$ from 3.1 dB for $4QC = 0$ to 1.6 dB for $4QC = 4$. Space charge had a negligible effect for lower magnitudes of phase velocity error, but for mm-wave and THz-regime devices which have appreciable-space-charge beams and potential for significant fabrication errors, these results illustrate that the fundamental forward-wave mode gain is less susceptible to fabrication errors.

Finally, we closely examined the effect of errors on the calculated 1-dB bandwidth of the maximum mean gain as shown in Figs. 6, 9, and 12. Again, we observe that impedance and loss perturbations have little effect (less than ±0.1% change in bandwidth for a 10% random variation). Phase velocity errors significantly impact the bandwidth however, and the effect is suppressed as $4QC$ is increased. For our example, a 10% phase velocity error ($\sigma_q = 0.1$) results in a reduction of the calculated 1-dB instantaneous bandwidth of 3.2% for $4QC = 0$, while for $4QC = 4$, we observe a reduction of only 1.4%. Again, we observe that, for this simulated TWT, having a higher space charge electron beam results in less sensitivity to fabrication errors.

V. Conclusion

We have expanded on the work summarized in [1] by exploring the effect of random fabrication errors on fundamental forward-wave mode gain in a TWT using an electron beam with finite space charge. We have also studied the impact of these errors on instantaneous 1-dB bandwidth. This was accomplished by reformulating the classic Pierce theory equations (including space charge) into a third-order differential equation which allowed us to introduce random perturbations to the Pierce parameters b, C, and d along the axial length of the TWT. These perturbations represent small axially-dependent variations to the RF phase velocity, interaction impedance, and loss, respectively. In an actual TWT, random or quasi-random fabrication errors on the critical dimensions of an SWS (i.e., the pitch of a helix SWS) can result in such perturbations to these parameters. This is particularly concerning for mm-wave and THz-regime sources and amplifiers where fabrication errors may represent a significant percentage of the critical SWS dimensions. In these cases, random fabrication errors may play a significant role in defining final device performance.

We observed that phase velocity errors have the largest effect on fundamental forward-wave gain and hence are the most important to control, similar to the conclusion drawn in [1]. This was true regardless of the amount of space charge present in the electron beam. When we looked solely at the phase velocity errors, we found that increasing the amount of space charge in the electron beam reduced the effect of fabrication errors. This effect was most significant for relatively large levels of phase velocity error (approximately 5% or greater) which remains relevant for mm-wave and THz-regime devices. Finally, we found that increasing the amount of phase velocity error resulted in a reduction to the instantaneous 1-dB bandwidth of the maximum mean gain for the design example studied. The bandwidth was not significantly affected by errors in the interaction impedance or RF loss.

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