Dual-Frequency, Harmonic, Magnetically Insulated Line Oscillator

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Abstract—Magnetically insulated line oscillators (MILOs) are cross-field devices which generate a self-induced azimuthal magnetic field via an axial current. This negates the need for external magnets, potentially increasing overall system efficiency at the expense of reduced device efficiency. This article reports the design, simulation, and experimental demonstration of a dual-frequency, harmonic MILO (HMILO), which is composed of two sequential slow wave structures (SWSs) tuned for oscillation at different frequencies, each paired with a set of choke cavities and insulated by the self-generated magnetic field of the common cathode. The two SWSs—designed for operation in L- and S-bands at 1 and 2 GHz, respectively—were tested independently before the HMILO experiments. Results for the L-band MILO (L-MILO) were reported previously by Packard.

In the experiments reported here, the isolated S-band MILO (S-MILO) produced $1.1 \pm 0.7$ MW of output power at $2.076 \pm 0.005$ GHz, when supplied with 207 kV and 7.3 kA from the MELBA-C generator. Ultimately, when implementing two separate extractor configurations, the HMILO produced $12.7 \pm 6.6$ MW at $0.984 \pm 0.013$ GHz and $3.2 \pm 1.5$ MW at $2.074 \pm 0.003$ GHz. These results are compared against simulated performance in the particle-in-cell (PIC) codes CST and improved concurrent electromagnetic PIC (ICEPIC).

Index Terms—Brillouin flow, cross-field, harmonic, high power microwaves (HPMs), magnetically insulated line oscillator (MILO), Marx-generators, pulsed power, radio frequency (RF), slow wave structure (SWS).

I. INTRODUCTION

HIGH-POWER microwaves (HPMs), focusing on a cross-field architecture, are generated through pulsed power (single or rep-rated pulses) or through continuous drive, and are of increasing interest due to their applications in communications, radar, plasma heating, commercial heating, medical treatment, plasma processing, and defense. Advances in HPM theory and practice benefit a variety of fields, including fusion, space exploration, and astrophysics [1], [2], [3], [4].

Utilization of HPM devices dates to Albert Hull’s magnetic valve design from 1921, which was developed to bypass a popular triode patent [5], [6], [7]. Hull inadvertently created what is now considered to be the first magnetron. Magnetrons became popularized in World War II by the Tizard Mission with their usage in the novel radar technology of the 1940s [7].

Within the last 30 years, research on a novel HPM technology called the magnetically insulated line oscillator (MILO) has become widespread, which shares some device similarities to the magnetron. The MILO device was patented by Clark et al. [8] and Bacon et al. [9] in 1987, with intended uses in fusion plasma heating, charged particle acceleration, and directed energy sources. The primary difference between MILOs and magnetrons is the requirement of an external magnetic field for a magnetron to achieve insulation of the electron hub. The magnetic field at which magnetic insulation is achieved is known as the Hull cutoff (HC) condition. The MILO, in contrast, produces self-magnetic insulation via axial flow of current along its cathode, which is unique among most cross-field devices.

In this work, we designed, simulated, and experimentally validated a high impedance, dual-frequency, harmonic MILO (HMILO) driven by a single source. The two frequency bands selected for the HMILO are the L-band (1 GHz) and the S-band (2 GHz). Nonharmonic dual-frequency MILOs, such as the BFMILO ($\approx 1.3$ and $\approx 1.5$ GHz) have been demonstrated previously [10]. For logical design progression, an L-band MILO (L-MILO) was developed and tested [11], followed by an S-band MILO (S-MILO), and culminating with the combination of the two slow wave structures (SWSs) in the HMILO. This harmonic design progression follows a related experiment performed by Packard et al. [12] on harmonic recirculating planar magnetrons.

Section II provides a brief background of the novel Brillouin flow solutions along with the application of such solutions for MILO design. Section III discusses simulation and
II. THEORY

The Brillouin flow solution for a cylindrical MILO was recently derived explicitly, in closed form, by Lau et al. [13]. This theory is formulated in terms of two parameters, the anode–cathode (AK) gap voltage \( V_a \) and the magnetic flux per unit axial length within the AK gap \( A_a \). These two quantities are normalized as

\[
\tilde{V}_a = \frac{V_a}{V_s} = \frac{Va}{V_s e/(mc^2)} \quad (1)
\]

\[
\tilde{A}_a = \frac{A_a}{A_s} = \frac{A_a e/(mc)}{\gamma} \quad (2)
\]

The gap voltage yields

\[
\gamma_a = \tilde{V}_a + 1 = \frac{1}{\sqrt{1 - \beta_a^2}} \quad (3)
\]

\[
\tilde{A}_a^{\min} = \gamma_a \beta_a \quad (4)
\]

where (4) gives the minimum normalized magnetic flux required for magnetic insulation at the given gap voltage, \( V_a \). Thus, the quantity

\[
f = \frac{\tilde{A}_a}{\tilde{A}_a^{\min}} \quad (5)
\]

then denotes the degree of magnetic insulation at a given magnetic flux, and \( f \) is roughly equal to the ratio of the magnetic field to the HC magnetic field [13]. Magnetic insulation requires \( f > 1 \). The two parameters, \( V_a \) and \( f \), then completely specify the Brillouin flow profile including the Brillouin flow hub height, flow velocity at the top of the Brillouin hub, the electron current carried within the Brillouin hub, and the anode (input) current, \( I_a \), that is required to yield this value of \( f \). Packard et al. [11] used this Brillouin flow theory to design and to interpret their MILO experiment operating at 240 kV and 10 kA at \( L \)-band. Here, we shall use a similar procedure to design the \( S \)-band oscillator (SBO), assuming that the \( L \)-band oscillator (LBO) and SBO are isolated from each other.

For an SBO operating at the \( \pi \)-mode of a SWS with a periodicity (pitch) \( P \), at a resonant frequency, \( f_{\text{res}} \), its phase velocity is given by

\[
v_{\text{ph}} = \frac{\omega}{k} = 2f_{\text{res}}P. \quad (6)
\]

Alternatively, specifying the frequency and the operating \( \pi \)-mode’s phase velocity (which is bounded by the Brillouin flow velocity at the top of the Brillouin hub), the pitch \( P \) is given by

\[
P = \frac{\beta_{\text{ph}} c}{2f_{\text{res}}}. \quad (7)
\]

The anode and cathode radius for the SBO were designed using the same procedure outlined by Packard et al. [11] for the \( L \)-band. A new formula on the degree of magnetic insulation at Buneman–Hartree (BH) condition is presented in the Appendix.

III. SIMULATION AND DESIGN

An intermediate step toward the HMILO was demonstration of successful, independent L-MILO and S-MILO. The L-MILO was already covered in-depth by Packard et al. [11], and the LBO portion of the HMILO utilizes the same geometric dimensions as Packard’s L-MILO. Therefore, design of only the S-MILO—and subsequently the SBO section of the HMILO—was required, with the L-MILO restricting some design scope.

As mentioned in Section I, target frequencies for the HMILO (and independent oscillators) are 1 GHz (LBO) and 2 GHz (SBO). As the SBO and LBO in the HMILO will be driven by the same pulsed power source, the current and voltage values observed in L-MILO experiments were used to guide the design. The source parameters used for dimensioning the (independent SBO) S-MILO were 250 kV and 10 kA [11]. Applying the theory, the radius of the cathode \( r_c \) was selected to be 8 mm, which gives the inner anode radius \( r_a \) of 25 mm, as shown in Fig. 1(b). These two radii are very similar to those reported for the L-MILO by Packard et al. [11]. This
simplifies manufacturing and assembly of the HMILO by keeping the anode and cathode radius the same. The radius of the outer anode \(a\) was calculated to be 63.75 mm (from cathode surface to the back of the vane, vane depth). The phase velocity \(\beta\) was set to 0.287 to align with the L-MILO’s design (see Fig. 2), which is identical to Packard et al. [11]. Therefore, the circuit pitch for the S-MILO SWS becomes 21.5 mm, according to (7).

Multiple simulation methods and tools were implemented alongside the analytic theory to design a dual-band HMILO. The main tools include: a unit cell approach using high frequency electromagnetic simulation software (HFSS from ANSYS) [14], an eigenmode solver for fast dispersion diagrams, a finite cavity method using CST Microwave Studio’s eigenmode solver for overall cold system design, and two particle-in-cell (PIC) codes for radio frequency (RF) e-beam (hot) system design. The PIC codes used are CST-Particle Studio (CST-PS) and improved concurrent electromagnetic PIC (ICEPIC) [15], [16], [17], [20].

A. HFSS Unit Cell

Initial operating parameters of both MILOs which were selected and determined to be feasible according to the theory. Unit cell simulation method was then employed in HFSS using hexahedral meshing and perfect electrical conductor boundaries to tune and improve the MILO design [18]. The unit cell method is a process of focusing in on a single pitch-length of a SWS of interest, forming a model of a single cavity [see Fig. 1(a)]. With this single cavity, one then applies a master/slave boundary condition to openings on each side of the SWS; this treats the system as an infinite structure along its axis [19]. Phase change then must be parameterized through at least one full period which mimics a transverse RF electric field wave propagating through the structure.

These HFSS unit cells provide fast and accurate cold tests and dispersion relation diagrams of each SWS [20]. In this case, the SWS cavities simulated are: 1–S-band cavity (SBO), 2–S-band choke cavity (SBO Choke), 3–L-band cavity (LBO), and 4–L-band choke cavity (LBO Choke), which are displayed in Fig. 1(a). The electric field is depicted for each unit cell at zero phase, representing fundamental mode operation. Overall placement of each unit cell in the full HMILO model is shown in Fig. 1(b).

Results of the unit cell simulations on individual cavities [see Fig. 1(a)] are presented in Fig. 2, which is a dispersion diagram displaying the frequency as a function of a normalized phase propagation constant \(\beta\) [21], [22]. The phase propagation constant is multiplied by the pitch of the SBO cavity \(P_{SBO}\) and divided by \(\pi\), simplifying the x-axis. The dispersion diagram is normalized to the SBO \(\pi\)-mode, in solid red, so the LBO \(\pi\)-mode, in solid blue, corresponds to a value of 0.5 on the horizontal axis. Both the fundamental mode (TM01) and next higher order transverse magnetic mode (TM02) are plotted for all four-unit cells, along with the corresponding beamline that interacts with the SBO and LBO cavity’s \(\pi\)-modes simultaneously, allowing spoke formation to occur synchronously in both structures. The phase velocity of the beamline for dual \(\pi\)-mode operation is 0.287c. Inclusion of 10% detuned beamlines (black dotted lines) in Fig. 2 demonstrate the spread in the electron’s axial beam velocities which is intrinsic in the Brillouin flow model. The next higher order modes that are supported by the LBO and SBO \(\pi\)-modes simultaneously, allowing spoke formation to occur synchronously in both structures. The phase velocity of the beamline for dual \(\pi\)-mode operation is 0.287c. Inclusion of 10% detuned beamlines (black dotted lines) in Fig. 2 demonstrate the spread in the electron’s axial beam velocities which is intrinsic in the Brillouin flow model. The next higher order modes that are supported by the LBO and SBO \(\pi\)-modes simultaneously, allowing spoke formation to occur synchronously in both structures. The phase velocity of the beamline for dual \(\pi\)-mode operation is 0.287c. Inclusion of 10% detuned beamlines (black dotted lines) in Fig. 2 demonstrate the spread in the electron’s axial beam velocities which is intrinsic in the Brillouin flow model. The next higher order modes that are supported by the LBO and SBO \(\pi\)-modes simultaneously, allowing spoke formation to occur synchronously in both structures. The phase velocity of the beamline for dual \(\pi\)-mode operation is 0.287c. Inclusion of 10% detuned beamlines (black dotted lines) in Fig. 2 demonstrate the spread in the electron’s axial beam velocities which is intrinsic in the Brillouin flow model.
occurred between the fundamental mode (TM_{01}), and the all the higher order modes (HEM_{11}, HEM_{21}, HEM_{31}, and TM_{02}), in what is often called the forbidden region of operation [26]. If a higher order mode does occur, it will not share the same frequency with the fundamental mode of the structure.

Selection of the ratio of vane to cavity width in terms of the pitch was tuned with unit cell simulation and was selected to be 50%, meaning both the cavity and vane thicknesses are 10.75 mm thick [27], [28], [29]. Table I lists key modes extracted from the dispersion relations in Fig. 2, which exhibit large separation of operational frequency between $\frac{3\pi}{5}$, $\frac{2\pi}{5}$, and $\frac{\pi}{5}$ modes for the fundamental wave mode in the primary SWS cavities (LBO and SBO). Minor separation was noted for the displayed three primary modes (flat top dispersion). The next higher order hybrid electric and magnetic modes at $\pi$-mode operation are also provided for convenience. Considerable detuning of the beam would be required to promote non-$\pi$-mode excitation in the SWS.

### B. CST Finite Model

With functional unit cells for the SBO and the LBO, each with a cavity and choke, a full finite cavity model was then generated with a merging of five cavities and two chokes on L-band and S-band. This forms the initial CST eigenmode solver setup for a finite cavity model of the HMILO in Fig. 1(b). This model used a tetrahedral mesh and perfect electrical conductor for the eigenmode simulation. Selection of five cavities and two chokes achieved optimal operation based on length restrictions, maximized oscillation, and mode optimization. The chokes cavities are evanescent to the SWS’s $\pi$-mode wave causing a reflection of backwards propagating waves [30]. Design and placement of the extractor, beam-dump, and downstream diode (DSD) are covered in Section IV [26]. Results of the finite cavity simulation are tabulated in Table II for the fundamental wave mode at $\pi$-mode for only the finite five cavity SWS of the SBO and LBO. All of the higher order modes frequency operation at $\pi$ mode agree with unit cell model from Table I.

### C. PIC Simulation

Computationally expensive and time-consuming CST-PS PIC and ICEPIC simulations, which was also used previously for the L-MILO [11], likewise enabled fine tuning of the HMILO. They gave preliminary insight into the HMILO operation demonstrated in Figs. 3–5. The CST-PS and ICEPIC simulation models were the same as the model used for the finite cavity eigenmode solver, depicted in Fig. 1. While simulations of the S-MILO were conducted, for the sake of brevity, only the HMILO PIC simulations are presented here. Both simulations used approximately the same number of total hexahedral mesh cells, as determined by convergence studies of the output power as described by Greenwood [31].

As shown in Fig. 3(b), both PIC simulations applied a steady-state input voltage of 250 kV with a 200 ns rise time,
Fig. 4. FFTs of the PIC simulation RF output voltage from (a) CST (dominant frequency = 2.05 GHz) and (b) ICEPIC (dominant frequency = 2.026 GHz). CST simulations produced very weak L-band oscillations, relative to S-band.

Fast Fourier transforms (FFTs) obtained from the two PIC codes (see Fig. 4), showed some discrepancies in both the S-band frequency and L-band amplitude. The S-band frequency was dominant at 2.05 and 2.026 GHz for CST-PS and ICEPIC, respectively. Relative to the S-band, the CST-PS FFT demonstrates very little oscillation of the L-band frequency target, while the ICEPIC simulations exhibit more pronounced L-band signals. The low L-band FFT response is correlated with unmatched total quality factors (Total $Q$, see Table II) of the L- and S-bands structures, which suggests that the S-band would out-compete the L-band in the TM$_{01}$, $\pi$-mode in simulation. Placement of the S-band structure in series between the L-band structure and the extractor could cause an attenuation/coupling issue in the PIC simulation. This reduced L-band behavior is not observed in experiment. A strong frequency response at 4 GHz in both CST and ICEPIC is observed in the FFTs, alluding to harmonic behavior of the S-band frequency in the PIC simulations.

Output power over the 250 kV input voltage time is shown in Fig. 5. Generation of the output power trace was achieved using the electric field signal at the output extractor face, via a Poynting vector power calculation [29]. Steady-state output power from CST-PS PIC is 0.8 MW and ICEPIC is 0.5 MW depicted in Fig. 5. The variance can be attributed to difference in input and DSD current between the PIC codes; CST’s currents are 25% greater than ICEPIC. Accounting for the variance in input current fair agreement between the PIC simulation is demonstrated, as was observed by Andreev et al. [32]. The number of cavities and chokes for each SWS was optimized for peak output power from both PIC codes.

### IV. Experimental Configuration

As was previously described two MILO experiments were investigated, one is the S-MILO configuration shown in Fig. 6, and the other is the HMILO configuration with both L-band and S-band SWSs depicted in Fig. 7. Both these configurations are driven by the Michigan Electron Long Beam Accelerator with Ceramic insulating stack (MELBA-C) [33]. MELBA-C is a Marx–Abramyan generator with a load-dependent output of 200–300 kV and 1–10 kA for 200–500 ns. Roughing vacuum was achieved with a scroll pump down to 10$^{-3}$ torr scale, enabling the use of a cryogenic pump which attained low 10$^{-6}$ torr scale.

#### A. S-MILO Setup

The experimental setup of the S-MILO is shown in Fig. 6. The MELBA output, and MILO input, is on the right-hand side of the figure. The cathode is made of 303 stainless steel and is wrapped in a sleeve of red velvet. While our L-MILO varied the cathode radius in the beam dump [11], the S-MILO uses a uniform cathode radius, $r_c$, of 8 mm (7 mm without the velvet). The cathode and a graphite beam dump comprise the DSD. This gap is adjusted by varying the cathode length, which changes the axial current and, thus, the magnetic field generated. The beam dump is grounded to the chamber through three azimuthally spaced quarter-wave stubs, which are placed at a set axial distance to pass the extracted RF. A tapered extractor, made of 6061 aluminum, gradually converts the 10 cm diameter beam dump to the 1.9 cm diameter inner conductor of the WR340 distributed field adapter (DFA340). The DFA340 converts from the TM$_{01}$ coaxial mode of the extraction taper to the TE$_{10}$ mode of the WR340 waveguide [34], where the RF output subsequently passes through a waveguide window, is measured by a calibrated directional coupler, and is absorbed by the load.
From the directional coupler the RF signal is attenuated and split, where the first split RF signal is used for direct waveform capture via an oscilloscope (Agilent 54855A), and the other split RF signal is sent to diodes (Agilent 8472B) before being processed by a separate oscilloscope for power measurements (Tektronix TDS 3054B).

The S-band SWS, shown in the middle of Fig. 6 is also made of 303 stainless steel, see Section III for the geometric dimensions. The whole assembly is aligned with dowels and clamped together with six stainless steel rods.

B. HMILO Setup

The experimental setup for the HMILO is provided in Fig. 7 and follows the S-MILO configuration with a few changes: the addition of an L-band SWS and choke cavities, a lengthened cathode to accommodate the extended SWSs, an external taper to allow packaging of all the cavities in the vacuum chamber, and the interchangeable DFA extractors. The DFA340 and DFA650 have optimal transmission bands of 1.77–2.15 GHz and 0.94–1.02 GHz, respectively, with $S_{21}$ between $-0.2$ and $-0.02$ dB [26], [34]. Consequently, neither mode converter is capable of handling the full frequency range of the HMILO, and they must be swapped between shots to properly measure the full range of frequencies emitted by the HMILO.

V. EXPERIMENTAL RESULTS

Three different sets of experimental results are demonstrated in this section from the two different experimental configurations presented in Section IV. The first dataset is from the S-MILO configuration, and the other two datasets are from the HMILO experimental configuration. These two datasets from the HMILO configuration were collected using the DFA340 or DFA650, and will be referred to as the HMILO-S and HMILO-L, respectively. For the HMILO-L, a 0.9–1.1 GHz bandpass filter was added immediately following the directional coupler. This filter rejected the signal produced by the SBO which was outside the operating band of the directional coupler and could not produce calibrated power measurements.

A. S-MILO Results

A representative shot from the 55-shot S-MILO series is provided in Fig. 8. This demonstrates the voltage, current, power, impedance, and frequency from the S-MILO validating its operation. The FFT and time frequency analysis (TFA) indicate most of the output power occurs at 2.074 GHz from 675 to 775 ns.


B. HMILO Results

Similar to the S-MILO data in Fig. 8, nominal HMILO results are demonstrated in Fig. 9 for each extractor style. Fig. 9(a) provides driver voltage, input current, load impedance, and output power for shot 18695, which was a representative shot from a 25-shot series utilizing the HMILO-S extractor configuration. As Fig. 9(b) shows, the dominant frequency is 2.074 GHz, with oscillations from 800 to 900 ns. Fig. 9(c) shows the operational behavior of shot 18734 from the 17-shot series. This was nominally the same as the shot in Fig. 9(a), but the extraction system was changed to the HMILO-L configuration to make calibrated L-band power measurements. These L-band signals are plotted in Fig. 9(d), where we see the dominant frequency is 0.985 GHz with the highest output power occurring from 775 to 875 ns.

Fig. 10 presents a combination of the FFTs from each HMILO extraction configuration. This implies dual-band frequency operation of the HMILO driven by a single source, in general consistent with the theory, design, and simulations of both the S-MILO and HMILO. Agreement between experiment and eigenmode simulations (unit cell and finite cavity) is also demonstrated, which suggests that both SWSs ran in the intended fundamental TM01 mode. These separate FFTs were generated using the two separate extractors referenced in Section IV-B and the beginning of Section V. The first extractor, HMILO-S, produced the SBO extracted FFT, while the second extractor, HMILO-L, produced the LBO filtered and extracted FFT.

High impedance MILOs, like the HMILO and S-MILO, may be inherently inefficient. Lower current MILOs produce less magnetic insulation, filling the AK gap with a larger electron hub when compared to more conventional MILOs that operate between 25 and 75 kA. This reduces the ability of electrons to transfer potential energy to the RF wave. Solutions to increasing the efficiency would be to operate at lower impedance/higher current, improve RF extraction, and reduce AK gap impurities.

For convenience, Table III tabulates the key experimental data from the three different datasets produced with the two experimental configurations. These tabulated parameters are extracted from the same data as the traces in Figs. 8 and 9. They will be used to compare with the theory in the next section.

VI. COMPARISON TO THEORY

Given the AK gap voltage $V_a$, input current $I_a$, and the anode and cathode radii $r_a$ and $r_c$, the MILO operating point can be predicted using the Brillouin flow model [13]. The procedure is illustrated in detail for the MILO experiment in Packard et al. [11], who also applied the same procedure to several MILO experiments performed elsewhere [35], [36], [37], [38], [39]. The operating point in these experiments, relative to the BH condition, is also determined, once the phase velocity of the operating mode is given. We shall use the same procedure for the data points in our dual-frequency MILO experiments, assuming that operation at each frequency is independent of the other. The operating points for our dual-frequency MILO (see Table III) are shown in Fig. 11, along with the prior MILO experiments [11], [35], [36], [37], [38], [39].

Table IV presents a summary of relevant design and operating parameters for the MILO designs presented here, as well as the several MILOs treated by Packard et al. [11], Haworth et al. [35], Eastwood et al. [36], Cousin et al. [37], Yu-Wei et al. [38], and Min et al. [39]. The operating point of these experiments is shown in Fig. 11. It gives the degree of magnetic insulation ($f$, see (5)) in Fig. 11(b), or the amount of magnetic flux within the AK gap ($\Phi_a$) in Fig. 11(a).

Note from Fig. 11 that our dual frequency MILO experiments operate much closer to the HC ($f = 1$) than the BH condition, and this trend is consistent with other MILO’s, as observed by Packard et al. [11] and shown in Fig. 11.
The BH condition usually occurs at $f > f_u$, where $f_u$ is a function of AK gap voltage only, and is given in [11], Fig. 3(a). Physically, $f_u$ is the value of magnetic insulation beyond which the anode current exceeds that required at HC ($f = 1$). The value of $f$ at the BH condition is given in (10) of the Appendix. It is shown in Fig. 11(b) for various experiments [11], [35], [36], [37], [38], [39].

MILO has the peculiar property that it can achieve magnetic insulation at an anode current lower than that required at HC [13]. This occurs for $1 < f < f_u$. In this range of $f$, the minimum anode current to achieve magnetic insulation occurs at $f_m$, which depends only on the voltage $V_a$ and is also shown in [11], Fig. 3(a)].

Note that both $f_u$ and $f_m$ are also displayed in Fig. 11(a) and (b). A representative sample of MILOs illustrated in Fig. 11, with the exceptions of Haworth et al. [35], Cousin et al. [37], S-MILO, and HMILO-S are within the v-shaped curve in the range of $1 < f < f_u$, assuming $f > f_u$ placing the MILO operation on the right-hand side of the curve. Moreover, they operate with magnetic insulation less than BH [11].

Like Cousin’s MILO, the S-MILO and HMILO-S experiments average operation did not achieve full magnetic insulation ($f < 1$). For the S-MILO at 207 kV, 7.3 kA was observed, while the required minimum current for magnetic insulation was 8.4 kA. Similarly, for the HMILO-S at 238 kV, 8.6 kA was observed and 9.2 kA was required for magnetic

### Table IV

<table>
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<th>Experiments and References</th>
<th>$r_a$ (cm)</th>
<th>$r_c$ (cm)</th>
<th>$r_a/r_c$</th>
<th>$v_p h/c$</th>
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<td>0.286</td>
<td>207</td>
<td>7.3$^1$</td>
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<td>0.286</td>
<td>238</td>
<td>8.6$^2$</td>
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$^1$ S-MILO requires 8.4 kA minimum current at 207 kV

$^2$ HMILO-S requires 9.2 kA minimum current at 238 kV

$^3$ Cousin’s MILO requires 31.6 kA minimum current at 400 kV
insulation. This places their operational points 10% below the theoretical minimum, $f_m$, as indicated in the footnote for Table IV. Consequently, this indicates the S-MILO and HMILO-S either oscillated without achieving complete magnetic insulation or barely achieved full insulation within the uncertainty bounds of the diagnostics utilized for the experiments. For these reasons, the S-MILO, HMILO-S, and the Cousin experiment intercept the $f_m$ curve in Fig. 11, but theoretically they fall in the range $f < 1$. The HMILO-L did operate slightly beyond magnetic insulation, with parameters similar to Packard’s L-MILO [11], as shown in Table IV and Fig. 11.

VII. CONCLUSION
Dual-frequency, harmonic oscillation at L-band (1 GHz) and S-band (2 GHz) was simulated and measured experimentally in a single MILO driven with a single driver, MELBA-C. Prior to the demonstration of the HMILO, an S-MILO, was also experimentally tested. The SWS of the S-MILO would subsequently become the S-band SWS of the HMILO. Brillouin flow theory was used for initial design and predictions of both S-MILO and HMILO behavior. Moreover, the novel theory agrees, within reasonable uncertainty, with the measured experimental behavior of the S-MILO and HMILO which operate at an anode current lower than that required for HC
methods of the RF signal from a MILO could improve/modify BH, $f$ of the gap voltage, $V$ where a $v$-shaped curve, specifically whether to right or left of $f_m$ ($f < f_m$ or $f > f_m$). Also, work on novel radial extraction methods of the RF signal from a MILO could improve/modify packaging of the system.

**APPENDIX**

**DEGREE OF MAGNETIC INSULATION ($f$) AT BH CONDITION**

This appendix presents a simple expression of the value of $f$, the degree of magnetic insulation defined in (5), when the BH condition is satisfied. This expression, given in (10) below, depends only on the phase velocity and on the gap voltage. It is independent of the mode and of the MILO geometry. Use of this expression readily shows that MILOs typically operate with a value of $f$ much closer to HC than BH, as shown in Fig. 11.

The BH condition reads in our notation (see [13], eq. (3.19)) or [11], eq. (15))

$$\tilde{V}_a = \tilde{A}_a \beta_{ph} + [\sqrt{1 - \beta_{ph}^2} - 1]$$

(8)

where $\beta_{ph}$ is the normalized phase velocity, $v_{ph}/c$. Equation (9) gives the value of $\tilde{A}_a$ at BH

$$\tilde{A}_a^{BH} = \frac{\gamma_a - \sqrt{1 - \beta_{ph}^2}}{\beta_{ph}} = \frac{\gamma_a \beta_{ph} - 1}{\gamma_{ph} \beta_{ph}}$$

(9)

where $\gamma_{ph} = 1/(1 - \beta_{ph}^2)^{1/2}$ and $\gamma_a$ is defined in (3) in terms of the gap voltage, $V_a$. The degree of magnetic insulation at BH, $f^{BH}$ follows from (4) and (5):

$$f^{BH} = \frac{\tilde{A}_a^{BH}}{A_a^{min}} = \frac{\gamma_a \beta_{ph} - 1}{(\gamma_{ph} \beta_{ph})(\gamma_a \beta_a)}$$

(10)

Note that $f^{BH}$ depends only on the phase velocity ($\beta_{ph}$) and on the gap voltage ($\gamma_a$). Fig. 12 plots $f^{BH}$ as a function of $V_a$ at various values of $\beta_{ph}$.

We make the following remarks on the use of (8)–(10) for MILOs, and for magnetrons.

1) Equation (8) is valid in general, regardless of how the magnetic insulation is provided. It is applicable to magnetron, MILO, and their hybrid.

2) While (8) was derived from the Brillouin flow model for a cylindrical MILO, the same BH condition was obtained from the single particle model.

3) If a MILO operates with a high phase velocity, such as $\beta_{ph} \approx 0.6$, the Brillouin flow model is unreliable, because it implies a Brillouin hub extending significantly into the AK gap (in order that the electron velocity at the hub edge has a high value of $\beta_{ph}$), i.e., close to HC. All crossed-field flows close to HC are sensitive to small perturbation.

4) For $\beta_{ph} < 0.2$, operation at BH condition implies good magnetic insulation.

Since (8) and (9) are valid for MILO, magnetron, and their hybrid [13], (10) is applicable to them.

**REFERENCES**


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