Theory of Traveling-Wave Tube Including Space Charge Effects on the Circuit Mode and Distributed Cold Tube Loss

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***Abstract*— This article presents a small-signal theory of a tape helix traveling-wave tube (TWT) with distributed cold tube loss, including the recently discovered space charge effects on the circuit mode that was obtained from the exact dispersion relation of the corresponding lossless tube. The classical values of Pierce's parameter C and Q are modified in this new formulation, while the cold tube loss will enter in a similar way as in Pierce. Realistic examples are given, including tubes with uniform circuit loss and with a spatial sever, and compared with the Pierce's classical theory.**

***Index Terms*— Dispersion relation, Pierce parameters, traveling-wave tubes (TWT).**

I. Introduction

IT WAS recently found that Pierce's classical theory of traveling-wave tube (TWT) [1] requires revision at a high electron beam current because of space charge effects. A new space charge parameter, q, was discovered that describes the modification of the circuit mode [2], [3]. It is analogous to Pierce's original space charge parameter, Q, which describes the modification of the beam mode. Pierce's three-wave TWT dispersion relation is thus modified to read [2]

$$\left[(\beta-\beta_e)^2-4\beta_e^2QC^3\right][\beta-\beta_p-4\beta_p qC^3]=-\beta_e^3C^3. \quad (1)$$

In (1), β is the propagation constant at frequency ω, $\beta_e = \omega/v_o$, $\beta_p = \omega/v_p$, v_0 is the beam velocity, v_p is the phase velocity of the forward circuit wave, and C is the gain parameter which determines the coupling between the beam mode [the first square bracket] and the circuit mode [the second square bracket]. It is seen from (1) that the presence of q in effect introduces a potentially significant detune in the circuit phase velocity, by $4qC^3$, which amounts to approximately two percent in a realistic example of a tape helix TWT [2].

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Setting $q = 0$ in (1) recovers Pierce's classical three-wave theory [1], [4].

The discovery of q originated from our study of the numerical solutions to the (very complicated) exact hot tube dispersion relation that was derived for a *lossless*, tape helix TWT model [3]. Over the frequency range of interest, the exact hot tube dispersion relation always yields three (3) roots for β. In order for a third-degree polynomial of β to yield those same exact three roots, it must be written in the form of (1), in which the three parameters, Q, q, and C may be determined numerically in terms of those three roots of β obtained from the exact dispersion relation [2], [3].

If the tape helix is lossy, all three roots of the exact hot tube dispersion relation are complex, leading to three complex parameters, Q, q, and C if one uses the procedure given in [2]. These complex parameters are cumbersome to interpret in physical terms. In this article, we explore an intuitive, physical way to model the combined effect of q and the cold tube loss; the latter could be uniform or nonuniform, as in a resistive sever.

II. Formulation

We propose that, when cold tube circuit loss is present, the modified Pierce dispersion relation should be

$$\left[(\beta-\beta_e)^2-4\beta_e^2QC^3\right]\left[\beta-\beta_p-4\beta_p qC^3+jCd\beta_e\right]=-\beta_e^3C^3 \quad (2)$$

where Q, q, and C are determined exactly *as if the circuit were lossless*, as in [2], and d is Pierce loss parameter associated with the cold tube loss. The cold tube propagation constant then reads, $\beta = \beta_p - jCd\beta_e$. The rationale for (2) follows. We want to be as close to Pierce's classical three-wave TWT theory as possible, both in the form of the dispersion relation and in its interpretation. Since this article concentrates on the further modification on the circuit mode due to cold-tube loss, the beam mode, which is represented by the first square bracket on the LHS of (2), remains unchanged. Next, we assume that the modifications to the lossless circuit mode, due to the space charge effect (modeled by q), and due to the circuit loss (modeled by d) are both small. Each of these two effects will enter linearly and independently in the circuit mode factor [second square bracket in (2)], and their mutual interaction becomes a higher order effect, which we ignore. Furthermore, we assume that the small correction due to the circuit loss would not change the coupling impedance that is represented by the RHS of (2), in which C has been calculated from the exact dispersion relation, so that this value of C is different from Pierce's theory (in which C was derived using only the dominant cold-tube circuit mode.) Equation (2), as written, then has three attractive properties: 1) the parameters Q, q, and C are all real because they were derived assuming $d = 0$; 2) (2) reduces to Pierce's original form (1) when $d = 0$, even though the parameters Q and C there now differ from that of Pierce's classical values; and 3) (2) reduces to the form of Pierce's classical theory for a lossy tube when $q = 0$ (though, again, not the values of Q and C).

While the dispersion relation (2) describes a uniform tube with the combined effect of q and d, we may generalize it to include distributed loss with $d = d(z)$. Note that this generalized description of $d(z)$ may include all sorts of cold tube loss, from the helix itself, from the support rods, and from some arbitrarily imposed loss profiles, by design. The spatial evolution along the tube axis (z) of the electronic displacement (s) and the normalized circuit electric field (a) is governed, respectively, by [5]

$$\left[\left(\frac{\partial}{\partial z} + j\beta_e\right)^2 + 4\beta_e^2 Q C^3\right] s = a \tag{3}$$

$$\left[\frac{\partial}{\partial z} + j\beta_p(1 + 4qC^3) + \beta_e C d\right] a = -j\beta_e^3 C^3 s. \tag{4}$$

When d is a constant, (3) and (4) admit simple exponential solution of the form $e^{-j\beta z}$ for both s and a, and the propagation constant β is readily shown to be governed by (2) for a uniform tube, whose gain (in dB) in the circuit wave power, $|a^2|$, at a distance L from the input is [4, eq. (11.1-15)]

$$G = 20 \log \left| \sum_{k=1}^{3} \frac{\delta_k^2 + 4QC}{(\delta_k - \delta_l)(\delta_k - \delta_m)} e^{\delta_k \beta_e C L} \right| \text{ dB.} \tag{5}$$

The three roots of β, $(\beta_k, \beta_l, \beta_m)$, to (2) are represented by δ, $(\delta_k, \delta_l, \delta_m)$, in (5) where $\delta_{k,l,m} = -j(\beta_{k,l,m} - \beta_e)/C\beta_e$. If there is a distributed loss, $d = d(z)$, the gain in the circuit wave power, $|a^2|$, must generally be computed numerically from (3) and (4).

III. Numerical Results

We shall adopt the same tape helix TWT model of Chernin *et al.* [6] and extend it to a lossy dielectric support layer. The beam and circuit parameters are the same as those in [2, Sec. III]; they are summarized in Table I. We assume that the dielectric layer has the same real part of the dielectric constant, $\epsilon_r = 1.25$; the imaginary part is specified by the loss tangent, $\tan(\delta) \equiv \epsilon_i/\epsilon_r$. The lossless tube corresponds to $\epsilon_i = 0$, for which the cold tube circuit mode propagation constant is designated as β_{p0}, which is real, and is calculated exactly as in [6]. A nonzero ϵ_i introduces an imaginary part, and a shift in the real part of the cold tube propagation constant. The percentage change of these real and imaginary parts is shown in Fig. 1 for a uniform tube with $\epsilon_i/\epsilon_r = 0.01$, 0.05, 0.1, 0.2. We need to retain the change in the real part because it yields a similar effect as q according to (2). However, a comparison of Fig. 1(a) and (b) shows that the change in the real part is much smaller than the change in the imaginary part of the cold tube propagation constant. We also find that this change in the real part of β_p (from β_{p0}) is also much smaller than the detuning effect due to q for our numerical examples [see (2)]. The loss parameter, d, is [1], [4]

$$d = \mathrm{Im}\left(\beta_p\right)/\beta_e C = 0.01836\Lambda/C \tag{6}$$

where Λ is the cold-tube loss in dB per axial wavelength of the beam mode.

TABLE I

Tape Helix Parameters for Test Case

Parameter	Value
Tape radius a	0.1245 cm
Helix pitch p	0.080137 cm
Pitch angle ψ	5.85°
Tape width w	0.0159 cm
Real dielectric constant of supporting area ϵ_r	1.25
Beam voltage V_b	3 kV
Beam current I_b	0.17 A
Beam radius R_b	0.05 cm

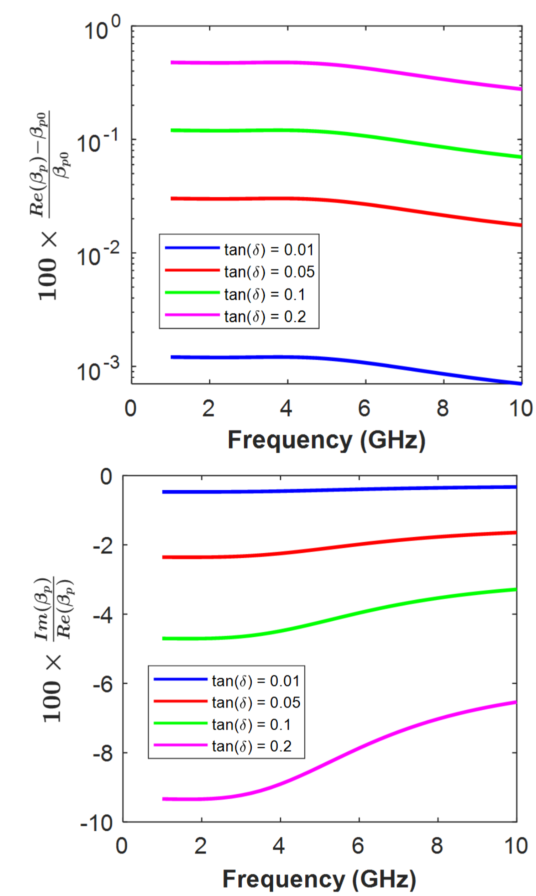

Fig. 1. (a) Percent difference between the real part of lossy $\boldsymbol{\beta_p}$ and the lossless $\boldsymbol{\beta_{p0}}$ (b) Ratio (in percent) between the imaginary and real components of lossy $\boldsymbol{\beta_p}$.

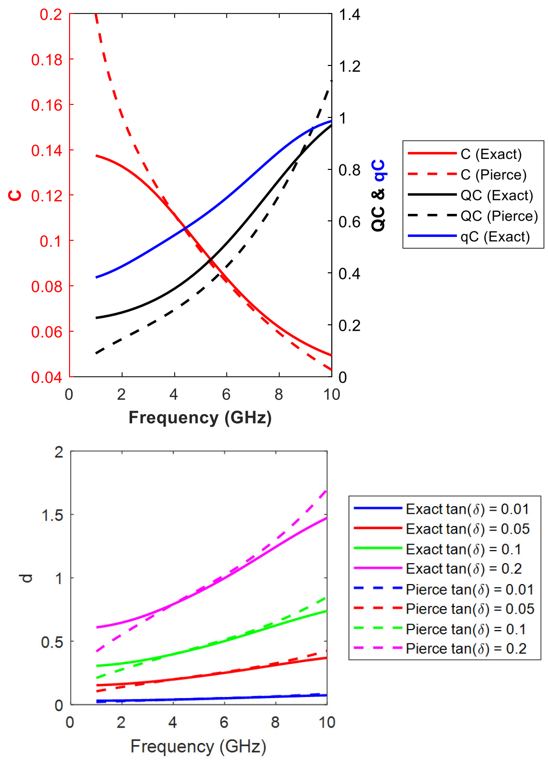

Fig. 2. Plots of Pierce parameters. (a) $\boldsymbol{C}$, $\boldsymbol{QC}$, and $\boldsymbol{qC}$ for $d = 0$. (b) $\boldsymbol{d}$ from the exact solution and the classical Pierce approach.

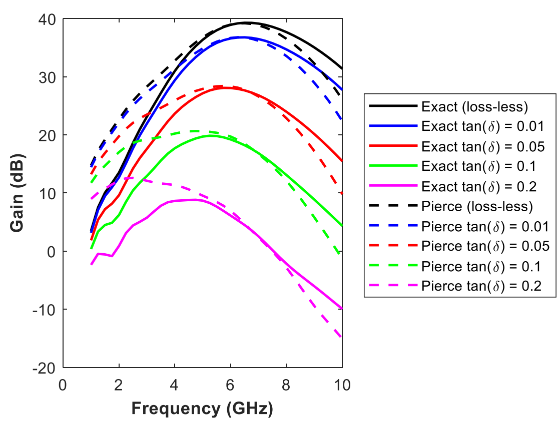

Fig. 3. Gain for a TWT of interaction length $\boldsymbol{L} = 10$ cm from exact and Pierce solutions.

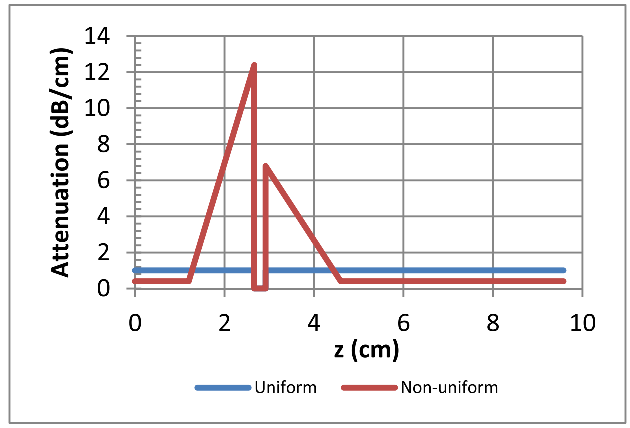

Fig. 4. Attenuation profiles of TWT with sever between $z_- = 2.667$ cm and $z_+ = 2.921$ cm. From [5].

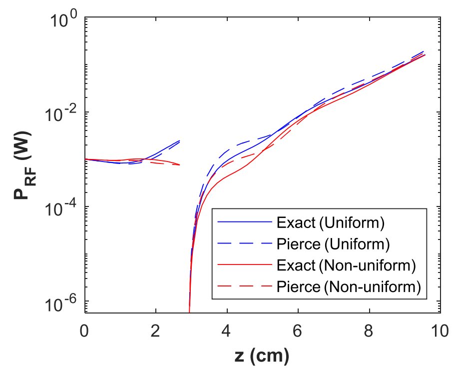

Fig. 5. RF power profile for the Pierce and exact cases using the uniform and nonuniform attenuation profiles at 4.5 GHz with input power of 1 mW.

In Figs. 2, 3, and 5, we compare the "exact" theory with the "Pierce" theory. By "exact" theory, we mean that Q, q, and C are obtained from the exact hot tube dispersion relation, assuming a lossless tube [2], [3], and this value of C is used in (6) to obtain d. By "Pierce" theory, we mean $q = 0$, QC is determined from the Branch-Mihran model [7], and $C^3 = K_0 I_b/(4V_b)$ where K_0 is given by [4, eqs. (10.3-22) and (10.2-21)]

$$K_0 \cong \frac{15\ \Omega}{ka} \times \frac{1}{S I_0^2(\gamma_0 a)} \int I_0^2(\gamma_0 r)\, dS \qquad (7)$$

where $k = \omega/c$, S is the beam cross-sectional area, $\gamma_0^2 = \beta_p^2 - k^2$, and $I_0(x)$ is the modified Bessel function of the first kind of zeroth order. This classical value of C is used in (6) for the "Pierce" theory. We emphasize that this classical value of C, which employs only the dominant cold-tube circuit mode, is different from the C obtained from the exact theory [2], as shown in Fig. 2(a). It is this exact value of C that is used in (2) which marks one of the departures from the classical Pierce's theory [the second departure is the value of Q, the third departure is the value of d because the value of d depends on the value of C, as shown in (6), and, finally, the fourth departure is the presence of q in (2)]. Fig. 2(b) shows the differences between the value of d in the "exact" theory and "Pierce" theory.

The gain for a tube with interaction length $L = 10$ cm, calculated using (5), is plotted in Fig. 3. The exact and Pierce theory agree well only in a restricted frequency range, and significant divergence is observed below 4 GHz and above 8 GHz. This is most likely attributed to the discrepancy in the same frequency range in C and in d (Fig. 2).

We next consider two realistic test cases of TWTs with severs [5]; one has uniform attenuation while the other has a variable attenuation profile (Fig. 4). Fixing the frequency at 4.5 GHz, we find that the attenuation in the uniform case corresponded to a loss tangent of 0.03. The attenuation profile, $d(z)$, for the nonuniform case in Fig. 4 may then be scaled accordingly (excluding the sever region, $z_- < z < z_+$).

The numerical integration of (3) and (4), including the sever region, are detailed in [5]. Fig. 5 shows general agreement between the exact and Pierce theory over the length of the tube, although significant divergence is observed immediately after the sever. This discrepancy is due to the detuning effect of q, although Q is compensated which leads to good agreement between the two solutions [see Fig. 2(a)]. We also observe higher gain in the uniform case immediately before and after the sever, illustrating the effects of increased attenuation in those regions in the nonuniform case.

IV. Concluding Remarks

This article describes a linear theory of a lossy tape helix TWT that includes the space charge effects on both the beam mode and in the circuit mode. The effects of uniform and nonuniform cold-tube loss is readily incorporated. This approach is being extended to the studies of backward wave oscillations in TWT [8], [9].

We have separately found that if the loss tangent in a uniform tube is less than 0.03 there is little difference in the gain whether we use $(\beta_k, \beta_l, \beta_m)$ obtained from (2) as we have done in this article, or the three complex roots of β that are obtained directly from the exact, hot tube dispersion relation [2], [3], with a complex ϵ.

Finally we note that when the beam voltage exceeds $\sim$10 kV a relativistic formulation is necessary in order to ensure that an accurate value of the beam velocity, and therefore of the detune factor $(\beta_e - \beta_p)$, is used in the dispersion relation. Otherwise the resulting error in the detune factor could become comparable or even greater than the detune effect due to q.

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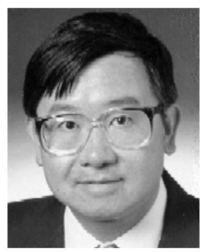

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